GRAND UNIFIED SUPERSYMMETRIC INFLATIONARY COSMOLOGY

By

MARTÍ RUIZ-ALTABA

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Per al Padrí

To all those who have died in the
struggle for justice while I wondered
about the mathematical consistency
of observable reality.

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Martí Ruiz-Altaba

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A theoretical laboratory is presented, within the framework of supergravity unification, which satisfies all experimental and cosmological constraints below the Planck scale, notably those from standard big bang nucleosynthesis, baryogenesis, and the fermionic spectrum. An intermediate scale, around $10^{10} - 10^{11}\,\mathrm{GeV}$, arises from the examination of inflation, supersymmetry breaking, the Peccei-Quinn symmetry, fermion masses, proton decay, and the baryon asymmetry of the universe. Careful attention is paid to the renormalization of proton decay operators. Although a realistic low energy spectrum follows from an isospin-like global symmetry (familiarity), a rather baroque higgs sector is needed to cancel anomalies and implement the missing multiplet mechanism. A low reheat temperature and baryogenesis from out-of-equilibrium decays are perhaps the most noteworthy aspects of the model, whose general success is not yet matched by any models derived from strings rather than point fields.

INTRODUCTION

One of the most astounding successes of the grand unification program,[1] initiated about ten years ago, was the understanding of the origin of the baryon asymmetry of the universe.[2] The program suffered an early setback when minimal $SU(5)$ failed to explain quantitatively this baryon asymmetry. It also predicted a fast proton decay rate which was soon ruled out by experiment. Supersymmetry[3,4,5] generally extends the proton's lifetime and prefers the strange and muonic decay modes. The experimental limits on the proton's lifetime[6] and the requirements of efficient baryogenesis tightly constrain the parameters in the Lagrangian of candidate unification models. An excellent theoretical laboratory arises from the locally supersymmetric grand unified model below,[7,8,9,10,11,12,13] which is consistent

a) internally below the Planck scale (anomaly-free),

b) with the standard $SU(3) \times SU(2) \times U(1)$ model at accelerator energies,

c) with big bang inflationary cosmology[14,15,16] at very early times, and

d) with underground detector limits on proton decay.

The general traits of this supersymmetric inflationary cosmology (SIC) are presented below. The seminal papers for SIC are Refs. 7 and 8. Further work appeared in Refs. 11 and 12. Although the model is not particularly aesthetic, it serves to prove, by construction, the consistency of all known constraints with simple supergravity unification.

Supersymmetry is spontaneously broken by the O'Raifeartaigh mechanism[17,18] and its breaking is related to the end of the inflationary period in the evolution of our universe. The cosmological constant can be set to zero

while supersymmetry is broken, but a contribution of $O(M_W^2 M_{\text{Planck}}^2)$ cannot be avoided after the observable sector finds out about supersymmetry breaking. No mechanism is known in any model to solve the cosmological constant problem, so we proceed with the rest of the program without much uneasiness.

A specific model with $SU(5)$ invariance is written down, relying freely on insights gathered from explorations with larger groups. The mass spectrum of leptons and quarks is incorporated at the tree level with the aid of couplings between matter and a nonminimal higgs sector including[19] a $\overline{\mathbf{45}}$. The element KM_{cb} of the quark mixing matrix has been measured from the (long) B-meson lifetime.[20] We incorporate it and a low top-quark mass (claimed and subsequently disclaimed by the UA1 group[21]), obtaining reasonable fermion mass relationships and adequate CP violation.

Since gravitinos are created during reheat,[22] the temperature T_R of the heat bath generating them is bounded above.[23] If reheated fields are thermalized immediately, the products of the reheating process are restricted to having masses $< T_R$, lest their abundance be strongly suppressed, $\sim \exp(-T_R/m)$. However, if the reheat is not an equilibrium process,[24] then no such strict bound can be derived for the reheated products, which could be more massive than T_R. In the SIC theory presented here, reheat is caused by the decay of inflatons, of mass around the intermediate scale, $10^{10} - 10^{11}$ GeV, and yet the reheat temperature is only about 10^6 GeV.

Higgs triplets of masses around the intermediate scale can be used for generating the baryon-number asymmetry[25,26,27] without causing fast proton decay via the dimension five higgsino-mediated operators.[28,29,30] The method presented here calls for right-handed neutrinos with masses around, again, the intermediate scale.

Proton decay rates, with largest branching ratios into strange and muonic modes, put a lower bound on the masses of the higgs triplets whose interactions violate baryon number. The requirement that these same triplets be produced from inflaton decay to generate the baryon asymmetry of the universe sets an upper bound on their masses. A narrow window satisfying these two bounds is found at the intermediate scale.

This dissertation is organized as follows. First, the SIC superpotential is presented and the hidden sectors are glossed over. The grand-unified (GUT) sector is then presented in detail along with its implications for fermion masses, CP violation, and neutrino oscillations,[31] in terms of the Yukawa couplings of the superpotential and the various symmetry-breaking scales. We then compute the decay rates for the proton, under flavor-helicity $SU(6)$, paying careful attention to the renormalization of the effective B-violating low-energy operators. Finally, we study baryogenesis and present a mechanism involving heavy right-handed neutrinos, whose masses are also around the intermediate scale.

The field content of supergravity[32,33] is as follows. Chiral superfields ($\overline{D}_\alpha \phi_i = 0$) contain matter, in spinorial and scalar varieties. Gauge superfields ($W_\alpha = \overline{D}^2 D_\alpha V,\ V = V^\dagger$) contain gauge vector bosons and the spinorial gauginos. In addition, the spectrum contains the gravity supermultiplet, with the graviton, the gravitino and the Kalb-Ramond field. Supergravity theories are never torsion-free due to the interdependence of the fermionic and bosonic coordinates of superspace.[34]

Minimal $N = 1$ supergravity in four dimensions has two arbitrary independent functions, which reduce to only one superpotential P in the "minimal"

case. Kinetic terms for all fields are in this case canonical, and the only requirement on P is that it be analytic, *i.e.*, function only of the superfields and not of their conjugate partners.

The superpotential for the SIC model consists of three sectors,[7]

$$P = I + G + S \tag{1.1}$$

where fields in one sector do not appear in the other two. Both I and S contain only gauge singlets, responsible for inflationary expansion and supersymmetry breaking respectively. The sector G contains all the usual matter, the higgs sector, and more.

The purely scalar potential for a minimal supergravity theory with superpotential P is given by

$$V_{scalar} = \exp\left\{\frac{1}{M^2}\sum|\phi_i|^2\right\}\left[\sum\left|\frac{\partial P}{\partial \phi_i} + \frac{\phi_i^\dagger P}{M^2}\right|^2 - \frac{3}{M^2}|P|^2\right] + \mathrm{D-terms} \tag{1.2}$$

where the sums run over all scalar (super)fields in the theory, denoted symbolically by ϕ_i, and M is the reduced Planck mass

$$M = \frac{1}{\sqrt{8\pi}} M_{Planck} = 2.4 \times 10^{18}\,\mathrm{GeV} \tag{1.3}$$

Cross-talk between the sectors is always suppressed by powers of M. Each of the three sectors interacts with the other two only gravitationally, so thermal equilibrium is possible only within each sector or as a consequence of fine-tuned initial conditions.

INFLATION

The inflation sector I of the superpotential can be constructed very simply[7] by assuming it contains only one chiral superfield, Φ. Scaling all fields by M and considering the scalar potential due to I alone,

$$V_I \;=\; e^{|\Phi|^2}\left[\left|\frac{\partial I}{\partial\Phi}+\Phi^\dagger I\right|^2 \;-\; 3|I|^2\right] \tag{2.1}$$

At the minimum of this potential (the true vacuum up to interference with other sectors), the cosmological constant should vanish, so $V_I(\Phi_0) = 0$. Requiring further supersymmetry to be unbroken at this minimum implies that the gravitino mass must be zero, which imples that

$$\left|\frac{\partial I}{\partial\Phi}+\Phi^\dagger I\right|_{\Phi=\Phi_0} \;=\; 0 \tag{2.2}$$

Hence

$$I(\Phi_0) \;=\; \frac{\partial I}{\partial\Phi}(\Phi_0) \;=\; 0 \tag{2.3}$$

A very simple I satisfying the above requirements is[7] a quadratic superpotential

$$I \;=\; \Delta^2(\Phi-\Phi_0)^2 \tag{2.4}$$

where Δ is a mass parameter setting the scale for inflation. This is the small parameter which appears in all succesful inflationary models,[2] fixed around $10^{-4}M$ to avoid the supersymmetry entropy crisis or Polonyi problem.[8,22] Without loss of generality, we can take the vacuum value of the inflaton, $\langle\phi\rangle = \Phi_0$, to be real.

Requiring the scalar potential to be flat at the origin,

$$\left(\frac{\partial V}{\partial \phi}\right)(0) \; = \; 0 \tag{2.5}$$

so that (new) inflation can occur with a slow rollover, sets $\Phi_0 = M = 1$. Letting the scalar component of the chiral superfield Φ be $\phi e^{i\theta}$, the scalar potential from this sector only is

$$V_I \; = \; \Delta^4 e^{\phi^2}\left[\phi^6 - 4\phi^5\cos\theta + (3+4\cos^2\theta)\phi^4 - 4\cos\theta\,\phi^3 + (3-4\cos^2\theta)\phi^2 + 1\right] \tag{2.6}$$

For any value of ϕ ("the radius") this potential has a minimum at $\theta = 0$. It is the only local minimum, so there is no θ-degeneracy and no domain walls may arise.

$$\begin{aligned} V_I(\phi) \; = \; V_I(\phi,\theta=0) \; &= \Delta^4 e^{\phi^2}\left(\phi^6 - 4\phi^5 + 7\phi^4 - 4\phi^3 - \phi^2 + 1\right) \\ &= \Delta^4\left(1 - 4\phi^3 + \frac{13}{2}\phi^4 - 8\phi^5 + \frac{23}{3}\phi^6 + O(\phi^7)\right) \\ &= 4e\Delta^4(\phi-1)^2 + 12e\Delta^4(\phi-1)^3 + O((\phi-1)^4) \end{aligned} \tag{2.7}$$

whence the mass of the inflaton is

$$m_\phi \; = \; 2\sqrt{e}\Delta^2 \tag{2.8}$$

Similarly,

$$V_I(\theta) \; = \; V_I(\theta,\phi=1) \; = \; 8e\Delta^4(1-\cos\theta) \tag{2.9}$$

so the mass of the angular field θ is

$$m_\theta \; = \; \sqrt{2}\,m_\phi \; = \; 2\sqrt{2e}\,\Delta^2 \tag{2.10}$$

The potential (2.7) is a standard inflationary potential, flat at the origin, with a slow descent towards the minimum at M, and a steep divergence for values of the field larger than M. The inflaton ϕ has cubic self-interaction

of strength $12\Delta^2$, and linear (gravitational) interactions with other sectors of strength $\sim \Delta^2$ as well. The inflaton will couple to fields in S and G, with a preference for the heaviest ones, and a decay rate

$$\Gamma_\phi \simeq (\Delta^2)^2 m_\phi \simeq \Delta^6 \tag{2.11}$$

At Planck time all particle species in the universe are maintained in thermal equilibrium by gravity. Regardless, however, of the initial conditions, each of the three sectors decouples from the others while gravity becomes weaker as the universe expands. Fields within each sector thermalize through Yukawa and gauge interactions, so the inflaton superfield effectively evolves alone in a thermal bath provided by S and G.

The inflaton will roll away[35] from its initial configuration[36,37]

$$\phi < H_0 \,, \quad \dot{\phi} \simeq 0 \tag{2.12}$$

destabilized by the (otherwise negligible) Hawking radiation term

$$\Delta V_H \simeq -\Delta^8 \phi \tag{2.13}$$

As the universe expands, the energy density becomes dominated by the potential energy of the inflaton, which can reasonably be assumed[38] to evolve according to the classical equations of motion

$$\ddot{\phi} + 3H\dot{\phi} + \frac{dV_I}{d\phi} = 0 \tag{2.14}$$

$$H^2 = \frac{1}{3M^2}\left(\frac{1}{2}\dot{\phi}^2 + V_I(\phi)\right) \tag{2.15}$$

where H is the Hubble constant (parameter, rather) with its value in anti-deSitter spacetime given by

$$H_0 = \sqrt{\frac{V(0)}{3M^2}} = \frac{\Delta^2}{\sqrt{3}M} \tag{2.16}$$

Inflation occurs with $\ddot{\phi} << 3H_0\,\dot{\phi}$ from a time t_* when $\phi \sim H_0$ until a time $t_I \sim m_\phi^{-1}$. The inflaton then starts evolving like matter, until a time $t_R \sim \Gamma_\phi^{-1}$ when it decays into radiation and reheats the universe.[7,8] Using conservation of energy, the increase in radiation energy density is determined by

$$\begin{aligned}
\rho_\phi(t_R) &= \rho_\phi(t_I)\left[\frac{t_I}{t_R}\right]^2 \\
&= \Delta^4\left[\frac{\Gamma_\phi}{m_\phi}\right]^2 \\
&\simeq \Delta^{12} \\
&\equiv \frac{\pi^2}{15}g^*T_R^4
\end{aligned} \tag{2.17}$$

with g^* the effective number of light degrees of freedom at the time of reheat t_R. The reheat temperature T_R for the heat bath in the causal bubble is, by definition,

$$T_R \simeq \Delta^3 \tag{2.18}$$

To avoid the monopole problem,[14,39] m_ϕ must be less than the unification scale$= M_X$; hence $\Delta^2 < 10^{-2}$. Below we shall find that actually $\Delta \sim 10^{-4}$, so the reheat temperature T_R is only about 10^6 GeV. Despite this low reheat temperature (which does not, by far, produce any problems with nucleosynthesis), inflatons are produced copiously and inflaton decay does generate enough baryon asymmetry, due to the lack of equilibrium among the various hidden sectors.

The number N_I of e-folds of inflationary expansion ($R_R \sim e^{N_I}R_I$) will be[9]

$$N_I = \int_{\phi_{initial}}^{\phi_{reheat}} H_0 dt \tag{2.19}$$

This is easily much larger than the observationally required $N_I \geq 65$. N_I is around 10^8 if $\phi_i < H_0$, $\phi_r \sim M/10$, so there is a large set of initial conditions leading to sufficient inflation.

In the deSitter (inflationary) stage, the quantum fluctuations[40,41,42,43] of the inflaton are of order H_0. Their comoving size leaves the horizon during inflation (Hubble radius is constant), and they reenter the horizon after inflation, thus generating a spectrum of density fluctuations at scale λ:

$$\frac{\delta\rho}{\rho}(\lambda) \simeq \frac{H_0^2}{\dot{\phi}^2(t_I)} \left[1 + \frac{3}{2}\log(\lambda H_0)\right]^2 \tag{2.20}$$

Approximating V_I by its truncated Taylor series around the origin, Eqs. (2.14) and (2.15) can be solved for $\phi(t_I)$ to give[7]

$$\frac{\delta\rho}{\rho} \sim 10^4\Delta^2 \sim 10^{-4} \tag{2.21}$$

which is the preferred value for galaxy formation with adiabatic density fluctuations, known in the trade as the "cold dark matter scenario." Interestingly, Hawking has shown[44] that if $\frac{\delta\rho}{\rho} < 10^{-4}$, then the mass of the inflaton must be less than 10^{15} GeV, or $\Delta < 10^{-1.5}$. Let us emphasize that, given the value of $\Delta \sim 10^{-4}$ required from the discussion of the gravitino problem,[8] the magnitude of the density fluctuations comes out as a prediction of the theory.

SUPERSYMMETRY BREAKING

Consider now the supersymmetry-breaking piece S of the superpotential P. It cannot be[22] of the Polonyi[45] type (linear in the superfield). Take S to be of the O'Raifeartaigh type,[8,17]

$$S \;=\; A(\lambda B^2 + \mu^2) + mBC + n \tag{3.1}$$

where A, B, C are chiral superfields and λ, μ, m, and n are parameters. We can set $m \simeq \mu$ and adjust n to ensure that the cosmological constant vanishes at the minimum of the full potential, taking into account the inflation and gauge sectors.

The F term $\frac{\partial S}{\partial A}$ breaks supersymmetry[17,18] so that the spinor component of A is the Goldstone fermion which provides the gravitino with the helicity $\frac{1}{2}$ states it needs to become massive. The scalar component of A, the O'Raifearton, couples weakly and stores potential energy during inflation. Then it evolves like matter until it decays preferentially into a gravitino pair through a one-loop diagram with three internal B propagators, with rate

$$\Gamma_{A\to\frac{3}{2}\frac{3}{2}} \;\simeq\; \frac{\alpha_\lambda^3}{16\pi^2}m_A \tag{3.2}$$

where $\alpha_\lambda = \lambda^2/4\pi$, and $m_A \simeq \alpha_\lambda\mu$ is the one-loop O'Raifearton mass.

The gravitino mass is $m_{\frac{3}{2}} = \mu^2/M$, and the B, C fields have a mass of order μ. For supersymmetry to solve the gauge hierarchy problem[46,47,48,49,50,51] (its precise *raison d'être*), $m_{\frac{3}{2}}$ must be at most around a TeV so $\mu \sim 10^{10-11}$ GeV $\sim (10^{-7} - 10^{-8})M$.

Now gravitinos couple to matter very weakly (with gravitational strength), so they are long-lived particles. They may thus decay into photons and photinos (lightest odd-R particles, $R = (-1)^{3B+L+2S}$) after the onset of nucleosynthesis. Their number density at decay is severely constrained[52,53,54] by the success of conventional nucleosynthesis[55] in explaining the primeval abundances of light element. The decay of gravitinos into gluon-gluino pairs, for instance, can dramatically alter the deuterium abundance.[56,57]

The abundance of gravitinos at the time of nucleosynthesis is constrained, whence the reheating temperature is bounded[24]

$$T_R \;<\; 2.5 \times 10^7\,\mathrm{GeV}\left[\frac{1\ \mathrm{TeV}}{m_{\frac{3}{2}}}\right] \tag{3.3}$$

Using $T_R \simeq \Delta^3$ and $m_{\frac{3}{2}} \simeq \mu^2$, this implies

$$\Delta^3\mu^2 < 4.2 \times 10^{-27} \tag{3.4}$$

which entails, under the requirement (for the solution of the hierarchy problem) that $\mu \simeq 10^{-8}$, that[8]

$$\Delta < 10^{-4} \tag{3.5}$$

If gravitinos happened to be stable, then the bound (3.3) is less restrictive,[24]

$$T_R \;<\; 1.3 \times 10^{11}\,\mathrm{GeV}\left[\frac{1\ \mathrm{TeV}}{m_{\frac{3}{2}}}\right] \tag{3.6}$$

implying

$$\Delta \;<\; 10^{-2.5} \tag{3.7}$$

To make sure that inflaton production of gravitinos is small, since the inflaton couples most strongly ($\sim \Delta^2$) to the heaviest fields, we must kinematically forbid the inflaton from decaying into O'Raifeartons, as well as B and C fields,

$$m_\phi \;<\; m_A, m_B\,, m_C \tag{3.8}$$

and require also α_λ to be small, $\alpha_\lambda < 10^{-3}$, in order to suppress one-loop effects.[8]

A careful analysis[8] of these one-loop effects reveals a lower bound for the inflation scale

$$\Delta < 10^{-4.2} \tag{3.9}$$

whence Δ is rather tightly constrained, much to nature's credit.

GRAND UNIFIED SECTOR

We write the superpotential for matter fields as[11]

$$
\begin{aligned}
G_1 &= \mathbf{F}^T \begin{pmatrix} 0 & a'H & 0 \\ aH & cL & 0 \\ 0 & 0 & bH \end{pmatrix} \mathbf{T} + \mathbf{T}^T \begin{pmatrix} 0 & dK & 0 \\ dK & eK' & fK'' \\ 0 & fK'' & gK \end{pmatrix} \mathbf{T} \\
&+ \mathbf{S}^T \begin{pmatrix} 0 & \tilde{d}_1 K & 0 \\ \tilde{d}_2 K & \tilde{e}K' & \tilde{f}_1 K'' \\ 0 & \tilde{f}_2 K'' & \tilde{g}K \end{pmatrix} \mathbf{F} + (\jmath_1 S_1 S_2 + \jmath_2 S_3 S_3)\, \phi_1
\end{aligned}
\tag{4.1}
$$

where $\mathbf{T}^T = (T_1, T_2, T_3)$, $\mathbf{F} = (F_1, F_2, F_3)$, and $\mathbf{S}^T = (S_1, S_2 S_3)$ are the $N = 1$ chiral superfields containing matter in three generations or families, and H, L, K, K' and K'' are higgs superfields, and ϕ_1 is a group singlet whose vacuum expectation value $\langle \phi_1 \rangle = M_0 = 10^{10}\,\mathrm{GeV}$ breaks the Peccei-Quinn symmetry[58].

The explicit field content of F_n, T_n, and S_n is shown below, where Greek indices run from one to three and denote color. All fields are chiral superfields with left-handed Weyl spinor components. The fields F_n are vectors, T_n antisymmetric tensors, and S_n singlets:

$$
\begin{aligned}
F_n &= \begin{pmatrix} (d_n^c)^\alpha \\ e_n \\ -\nu_n \end{pmatrix} \\
T_n &= \frac{1}{\sqrt{2}} \begin{pmatrix} \epsilon_{\alpha\beta\gamma}(u_n^c)^\gamma & -u_{n\alpha} & -d_{n\alpha} \\ & 0 & -e_n^c \\ & & 0 \end{pmatrix} \\
S_n &= \nu_n^c
\end{aligned}
\tag{4.2}
$$

The representations and global symmetry charges of the various fields in G_1 are shown in Table I. The only unusual feature is the higgs field L, which

transforms like a $\overline{\mathbf{45}}$. The first two terms in (4.1) constitute the Georgi-Jarlskog model[19] with an extra $eK'T_2T_2$ coupling, needed to correct the prediction $KM_{cb} \simeq \sqrt{\frac{m_c}{m_t}}$, incompatible with the value inferred from measurements of the B-meson lifetime[20] ($KM_{cb} = 0.0435 \pm 0.0003$) and the small top-quark mass ($< 150\,\mathrm{GeV}$) required by unitarity.

TABLE I. Field content and symmetries (vectorial, Peccei-Quinn, familiarity) of G_1.

Superfield	SU(5) irrep	V	X	$\mathcal{F}$
F_1	$\overline{\mathbf{5}}$	-3	1	1
T_1	$\mathbf{10}$	1	1	1
S_1	$\mathbf{1}$	5	1	1
F_2	$\overline{\mathbf{5}}$	-3	1	-1
T_2	$\mathbf{10}$	1	1	-1
S_2	$\mathbf{1}$	5	1	-1
F_3	$\overline{\mathbf{5}}$	-3	1	0
T_3	$\mathbf{10}$	1	1	0
S_3	$\mathbf{1}$	5	1	0
H	$\overline{\mathbf{5}}$	2	-2	0
L	$\overline{\mathbf{45}}$	2	-2	2
K	$\mathbf{5}$	-2	-2	0
K'	$\mathbf{5}$	-2	-2	2
K''	$\mathbf{5}$	-2	-2	1
ϕ_1	$\mathbf{1}$	-10	-2	0

The four phase symmetries can be understood as follows.[11] First, there is the vectorial symmetry V, which becomes $B-L$ below the electroweak breaking

scale. It forbids unwanted terms like FK and FFT and has no anomalies. Second, we have the anomalous Peccei-Quinn symmetry X, needed to solve the strong CP problem.[59] Third is the chiral R symmetry of cubic superpotentials, broken by gravitational couplings to the hidden sectors I and S. Lastly, we have the $\mathcal{F}$ symmetry, familiarity, which is the only one to distinguish among the three families or generations. Familiarity $\mathcal{F}$ is like family isospin, traceless in family space.

Crucially, we forbid[11] mixing terms between the two sets of higgs fields with different V, such as HK, HK', HK'' or $L\Sigma K$, $L\Sigma K'$, $L\Sigma K''$, where $\Sigma \sim \mathbf{24}$ or $\mathbf{75}$. Such mass terms would lead to disastrously fast proton decay rates, through dimension-five operators (see the discussion below on proton decay).

As a further and rather innocent simplification, we shall assume that the matrices are symmetric, *i.e.*, that $a = a'$, $\tilde{d}_1 = \tilde{d}_2$, and $\tilde{f}_1 = \tilde{f}_2$.

Fermion Masses

The higgs superfields produce after $SU(5)$ breaking light ($\sim M_W$) doublets and heavy ($\sim M_X^2/M$) triplets, via the missing multiplet mechanism.[60,61] Our purpose now is to determine the fermion masses, in order to constrain the various Yukawa parameters appearing in the superpotential. The $\Delta I_W = \frac{1}{2}$ masses come from the $SU(2)_L \times U(1)_Y \rightarrow U(1)_{\rm em}$ phase transition, when neutral components of higgs $SU(2)_L$ doublets acquire a vacuum expectation value (*vev*) σ. The $\Delta I_W = 1$ masses come from the breaking of the Peccei-Quinn phase symmetry, when some singlets acquire *vev*'s M_0. Rewriting (4.1) in terms of the higgs $(\mathbf{3,1})$ triplets and $(\mathbf{1,2})$ doublets, we arrive at the

$SU(3) \times SU(2)_L \times U(1)_Y$ superpotential for this sector, with due care for the normalization of the kinetic terms of the component fields.

The notation $\psi\chi$ is shorthand for $\psi^T C\chi$, with C the charge-conjugation matrix and T the transposition in Dirac space. A useful identity is

$$\left(\overline{\psi}_L^c\right)^T C \;=\; \psi_R \tag{4.3}$$

Since all spinors are left-handed,

$$\psi^T C\chi \;=\; \psi_L^T C\chi_L \;=\; \overline{\psi}_R^c \chi_L \;=\; \chi_L^T C\psi_L \;=\; \overline{\chi}_R^c \psi_L \tag{4.4}$$

In the more convenient two-component notation, in which

$$\psi^c \;\rightarrow\; \sigma_2 \psi_R^* \tag{4.5}$$

the various products are of one of the forms below:

$$\begin{aligned}
\psi\chi \;&\rightarrow \psi_L^T \chi_L \\
\psi^c\chi^c \;&\rightarrow (\sigma_2\psi_R^*)^T(\sigma_2\chi_R^*) \;=\; -\psi_R^\dagger\chi_R^* \;=\; -(\psi_R^T\chi_R)^* \\
\psi^c\chi \;&\rightarrow (\sigma_2\psi_R^*)^T\chi_L \;=\; -\psi_R^\dagger\sigma_2\chi_L \;=\; -\overline{\psi}_R\chi_L \\
\psi\chi^c \;&\rightarrow \psi_L^T(\sigma_2\chi_R^*) \;=\; -(\psi_L^\dagger\sigma_2\chi_R)^* \;=\; -(\overline{\psi}_L\chi_R)^*
\end{aligned} \tag{4.6}$$

corresponding to the left Majorana mass term, the right Majorana mass term, and the two pieces of the Dirac mass term. The mass Lagrangian for matter fermions is thus

$$\begin{aligned}
\mathcal{L} \;=&\; (d^c, s^c, b^c) M_{-\frac{1}{3}} \begin{pmatrix} d \\ s \\ b \end{pmatrix} \;+\; (u^c, c^c, t^c) M_{\frac{2}{3}} \begin{pmatrix} u \\ c \\ t \end{pmatrix} \;+\; (e^c, \mu^c, \tau^c) M_\ell \begin{pmatrix} e \\ \mu \\ \tau \end{pmatrix} \\
&+ (\nu_e^c, \nu_\mu^c, \nu_\tau^c) M_D \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} \;+\; (\nu_e^c, \nu_\mu^c, \nu_\tau^c) M_R \begin{pmatrix} \nu_e^c \\ \nu_\mu^c \\ \nu_\tau^c \end{pmatrix} \;+\; h.c.
\end{aligned} \tag{4.7}$$

The mass matrices are given below in terms of the parameters in the superpotential and the *vev*'s of the various doublets. It is clear that the first four

terms in (4.7) are Dirac masses of the form $\overline{\psi}_R\psi_L + h.c.$ In contrast, the last term is a "right-handed" Majorana mass of the form $\overline{\psi}^c_R\psi_L$.

The quark mass matrices are

$$M_{-\frac{1}{3}} = -\frac{\rho}{\sqrt{2}} \begin{pmatrix} 0 & a\langle H_2\rangle & 0 \\ a\langle H_2\rangle & \frac{c}{\sqrt{6}}\langle L_2\rangle & 0 \\ 0 & 0 & b\langle H_2\rangle \end{pmatrix} \tag{4.8}$$

$$M_{\frac{2}{3}} = -4\rho \begin{pmatrix} 0 & d\langle K_2\rangle & 0 \\ d\langle K_2\rangle & e\langle K'_2\rangle & f\langle K''_2\rangle \\ 0 & f\langle K''_2\rangle & g\langle K_2\rangle \end{pmatrix} \tag{4.9}$$

The parameter ρ represents an $SU(3)$ renormalization group enhancement factor between the GUT scale and low accelerator energies. The parameter ρ is determined by any one well-known ratio of eigenvalues of $M_{-\frac{1}{3}}$ and M_ℓ at low energies (see below), neglecting differences in the $SU(2)$ and $U(1)$ renormalization of the fermion masses.

The lepton mass matrices are

$$M_\ell = -\frac{1}{\sqrt{2}} \begin{pmatrix} 0 & a\langle H_2\rangle & 0 \\ a\langle H_2\rangle & \frac{3c}{\sqrt{6}}\langle L_2\rangle & 0 \\ 0 & 0 & b\langle H_2\rangle \end{pmatrix} \tag{4.10}$$

$$M_D = -\begin{pmatrix} 0 & \tilde{d}\langle K_2\rangle & 0 \\ \tilde{d}\langle K_2\rangle & \tilde{e}\langle K'_2\rangle & \tilde{f}\langle K''_2\rangle \\ 0 & \tilde{f}\langle K''_2\rangle & \tilde{g}\langle K_2\rangle \end{pmatrix} \tag{4.11}$$

$$M_R = \begin{pmatrix} 0 & j_1M_0 & 0 \\ j_1M_0 & 0 & 0 \\ 0 & 0 & j_2M_0 \end{pmatrix} \tag{4.12}$$

Now the two doublets from H and L (coupling to the $\overline{\mathbf{5}}\times\mathbf{10}$ sectors) will mix with doublets from other heavy fields, leaving light only the linear combination $\cos\alpha\, H_2 + \sin\alpha\, L_2$. Similarly, the only doublet from higgses coupling to the $\mathbf{10}\times\mathbf{10}$ sector which remains light is

$$\alpha_1 K + \alpha_2 K'_2 + \alpha_3 K''_2 \tag{4.13}$$

Inverting the unitary mixing matrices, we can express the *vev*'s in the mass matrices (4.8)-(4.12) in terms of the *vev*'s σ_{FT} and σ_{TT} of the light higgs doublets:

$$\begin{aligned} \langle H_2 \rangle &= \sigma_1 = \cos\alpha\, \sigma_{FT} \\ \langle L_2 \rangle &= \sigma_2 = -\sin\alpha\, \sigma_{FT} \\ \langle K_2 \rangle &= \sigma_3 = \alpha_1^\dagger\, \sigma_{TT} \\ \langle K_2' \rangle &= \sigma_4 = \alpha_2^\dagger\, \sigma_{TT} \\ \langle K_2'' \rangle &= \sigma_5 = \alpha_3^\dagger\, \sigma_{TT} \end{aligned} \tag{4.14}$$

with

$$\sum_{i=1}^{5} |\sigma_i|^2 = \sigma_0^2 = \frac{M_W^2}{4g^2} = (188\,\mathrm{GeV})^2 \tag{4.15}$$

We have enough freedom to redefine the righjt- and left-handed fields so that we can make the α_i real by rotating their phases away.

Proceed now to find the eigenvalues of the above mass matrices and the mass eigenstates ψ_m as a linear combination for the current eigenstates ψ which we have dealt with so far,

$$\psi = V\,\psi_m \tag{4.16}$$

with V unitary.

For charged fermions, all mass terms are Dirac, of the form $(+h.c.)$

$$\overline{\psi}_R M \psi_L = \overline{\psi}_R V V^{-1} M V V^{-1} \psi_L = \overline{\psi}_{mR} \Delta \psi_{mL} \tag{4.17}$$

where $\Delta = V^{-1}MV$ is diagonal. These mass eigenvalues can be made real and positive by further rotating the charge-conjugate ("right-handed") fields by some phases. Indeed, right-handed and left-handed fields mix differently in general.

For charge-$(-\frac{1}{3})$ fermions gettting their mass from σ_{FT},

$$V_\ell = \begin{pmatrix} \cos\theta_\ell & \sin\theta_\ell & 0 \\ -\sin\theta_\ell & \cos\theta_\ell & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{4.18}$$

and

$$V_{-\frac{1}{3}} = \begin{pmatrix} \cos\theta_c & \sin\theta_c & 0 \\ -\sin\theta_c & \cos\theta_c & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{4.19}$$

so that

$$\begin{aligned} \Delta_\ell &= diag(-m_e, m_\mu, m_\tau) \\ &= diag\left(-\frac{1}{\sqrt{3}}\frac{(a\sigma_1)^2}{c\sigma_2}, \frac{3}{\sqrt{12}}c\sigma_2, -\frac{1}{\sqrt{2}}b\sigma_1\right) \end{aligned} \tag{4.20}$$

and

$$\begin{aligned} \Delta_{-\frac{1}{3}} &= diag(m_d, -m_s, m_b) \\ &= diag\left(\rho\sqrt{3}\frac{(a\sigma_1)^2}{c\sigma_2}, \frac{\rho}{\sqrt{12}}c\sigma_2, -\frac{\rho}{\sqrt{2}}b\sigma_1\right) \end{aligned} \tag{4.21}$$

with

$$\cos 2\theta_c = \frac{m_s - m_d}{m_s + m_d}, \; i.e., \; \theta_c \simeq \sqrt{\frac{m_d}{m_s}} \tag{4.22}$$

and

$$\cos 2\theta_\ell = \frac{m_\mu - m_e}{m_\mu + m_e}, \; i.e., \; \theta_\ell \simeq \sqrt{\frac{m_e}{m_\mu}} \tag{4.23}$$

The Georgi-Jarlskøg relationships[19] are

$$\frac{m_e}{m_\mu} = \frac{1}{9}\frac{m_d}{m_s} \tag{4.24}$$

$$\frac{m_\mu}{m_\tau} = 3\frac{m_s}{m_b} \tag{4.25}$$

and we can identify the Yukawa couplings a, b, c, as well as the renormalization factor ρ:

$$\begin{aligned} a &= \frac{1}{\sigma_1}\sqrt{2m_e m_\mu} \\ b &= -\frac{\sqrt{2}}{\sigma_1}m_\tau \\ c &= \frac{2}{\sqrt{3}\sigma_2}m_\mu \end{aligned} \tag{4.26}$$

$$\rho = \frac{m_b}{m_\tau} = 2.81 \tag{4.27}$$

To make masses positive, let

$$V_{\ell L} = V_\ell \,, \quad V_{-\frac{1}{3}L} = V_{-\frac{1}{3}} \tag{4.28}$$

and redefine the charge-conjugate fields such that

$$V_{\ell R} = V_\ell \begin{pmatrix} -1 & & \\ & 1 & \\ & & 1 \end{pmatrix} \,, \quad V_{-\frac{1}{3}R} = V_{-\frac{1}{3}} \begin{pmatrix} 1 & & \\ & -1 & \\ & & 1 \end{pmatrix} \tag{4.29}$$

For the charge-$\frac{2}{3}$ quarks, which get their masses from σ_{TT}, first disregard entries in $M_{\frac{2}{3}}$ proportional to d ($<< e, f, g$), which means disregard the up-quark mass. There is then enough freedom to absorb all phases in $M_{\frac{2}{3}}$ by redefining the c- and t-quark fields (and their charge-conjugates). Assume, equivalently, that $e\sigma_4$, $d\sigma_5$, $g\sigma_3$ are real. Then

$$V_{\frac{2}{3}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\eta & \sin\eta \\ 0 & -\sin\eta & \cos\eta \end{pmatrix} + O(\frac{m_u}{m_c}) \tag{4.30}$$

with

$$\eta \simeq \frac{f\sigma_3}{g\sigma_3 - e\sigma_4} << 1 \tag{4.31}$$

so that

$$\begin{aligned} \Delta_{\frac{2}{3}} &\simeq diag[0, m_c, m_t] \\ &\simeq -4\rho \; diag[0, e\sigma_4 - 2\eta f\sigma_5, g\sigma_3 + 2\eta f\sigma_5] \end{aligned} \tag{4.32}$$

The Yukawa couplings are thus

$$\begin{aligned} e &= \frac{-m_c}{4\rho\sigma_4} \\ f &= \frac{\eta(m_t - m_c)}{4\rho\sigma_5} \simeq -e\frac{\sigma_4}{\sigma_5} \\ g &= -\frac{m_t + m_c}{4\rho\sigma_3} \end{aligned} \tag{4.33}$$

Reintroducing now the entry $d\sigma_3$, complex with phase $\frac{1}{2}\delta$, we can find CP violation in the full mixing matrix $V_{\frac{2}{3}}$. First examine the determinant of the mass matrix

$$\det M_{\frac{2}{3}} = -e^{i\delta} m_u m_c m_t = -(d\sigma_3)^2(g\sigma_3) \tag{4.34}$$

where $e^{i\delta}$ is the determinant of $V_{\frac{2}{3}L}$. The full mixing matrix is then

$$V_{\frac{2}{3}} \simeq \begin{pmatrix} \cos\theta_u & \sin\theta_u\, e^{i\frac{\delta}{2}} & -\eta\sin\theta_u\,\frac{m_c}{m_t}\,e^{i\frac{\delta}{2}} \\ -\sin\theta_u & \cos\theta_u\, e^{i\frac{\delta}{2}} & -\eta+\frac{m_c}{m_t}\,e^{-i\frac{\delta}{2}} \\ -\eta\sin\theta_u & -\frac{m_c}{m_t}+\eta\, e^{i\frac{\delta}{2}} & 1 \end{pmatrix} \tag{4.35}$$

where

$$\sin\theta_u \simeq \theta_u \simeq \sqrt{\frac{m_u}{m_c}} \tag{4.36}$$

and

$$\Delta_{\frac{2}{3}} = diag[-m_u, m_c, m_t] \tag{4.37}$$

with m_c and m_t given by (4.32) and

$$m_u = -2\rho\frac{|d\sigma_3|^2}{e\sigma_4} \tag{4.38}$$

so that

$$|d\sigma_3| = \frac{1}{4\rho}\sqrt{m_u m_c} \tag{4.39}$$

Letting

$$V_{\frac{2}{3}L} = V_{\frac{2}{3}}\,, \quad V_{\frac{2}{3}R} = V_{\frac{2}{3}}\begin{pmatrix} -1 & & \\ & 1 & \\ & & 1 \end{pmatrix} \tag{4.40}$$

and recalling that the hadronic weak current is

$$\begin{aligned} J_\mu &= \overline{\psi}_{m\frac{2}{3}}\gamma_\mu\frac{1-\gamma_5}{2}(KM)\psi_{m-\frac{1}{3}} \\ &= \overline{\psi}_{\frac{2}{3}R}\,\gamma_\mu\,\psi_{-\frac{1}{3}L} \end{aligned} \tag{4.41}$$

one can check that the Kobayashi-Maskawa matrix,[62] KM, ends up being

$$\begin{aligned} KM &= V^\dagger_{\frac{2}{3}R}\,V_{-\frac{1}{3}L} \\ &\simeq \begin{pmatrix} \cos\theta_c & \sin\theta_c & 0 \\ -\sin\theta_c\,\cos\eta & \cos\theta_c\cos\eta & \sin\eta \\ \sin\theta_c\,\sin\eta & -\cos\theta_c\,\sin\eta & \cos\eta \end{pmatrix} \end{aligned} \tag{4.42}$$

up to CP-violating effects, better studied with the help of the determinant (4.34).

This prediction is to be compared with the KM matrix obtained from an analysis of experimental data, which yields[63]

$$KM = \begin{array}{c} \\ u \\ c \\ t \end{array} \begin{array}{c} \begin{array}{ccc} d & s & b \end{array} \\ \begin{pmatrix} 0.9733 \pm 0.0024 & 0.231 \pm 0.003 & < 0.0052 \\ 0.24 \pm 0.03 & 0.97 \pm 0.007 & 0.0435 \pm 0.003 \\ \sim 0 & < 0.076 & \sim 0.999 \end{pmatrix} \end{array} \tag{4.43}$$

The best fit of (4.42) to (4.43) provides[11]

$$\begin{aligned} \theta_c &= (13.33 \pm 0.16)^\circ \\ \eta &= (2.49 \pm 0.01)^\circ \end{aligned} \tag{4.44}$$

so that we can establish (in disagreement with the usual Fritzsch mass matrix result,[64,65] and believing the top mass to lie[21] around 45 GeV) that

$$\eta \simeq \frac{m_c}{m_t} \tag{4.45}$$

We can now bound the Yukawa couplings involved, because it follows from (4.15) that $|\sigma_i| < 188\,\text{GeV}$. We use the values of ρ and η given by (4.27) and (4.31), the standard masses for charged leptons, and masses[66] of (5 Mev, 1.84 GeV, $45t$ GeV), where t is a dimensionless parameter assumed by certain wishful propagandists close to one.

$$\begin{aligned} |a| &> 5.5 \times 10^{-5} \\ |b| &> 1.3 \times 10^{-2} \\ |c| &> 6.5 \times 10^{-4} \\ |d| &> 4.5 \times 10^{-5} \\ |e|, |f| &> 8.7 \times 10^{-4} \\ |g| &> 2.1 \times 10^{-2}\, t^2 \end{aligned} \tag{4.46}$$

To find the eigenvalues and eigenmasses for neutrinos (left-handed ones are naturally light due to the see-saw mechanism[67,68]), we must diagonalize

a 6×6 matrix which takes a particular form, with the upper-left 3×3 block equal to zero. Following the construction below,[69] we can reduce the problem to the diagonalization of two 3×3 matrices instead.

In the basis $(\nu_e, \nu_\mu, \nu_\tau, \nu_e^c, \nu_\mu^c, \nu_\tau^c)$, the full mass matrix is

$$M = \begin{pmatrix} 0 & M_D \\ M_D^T & M_R \end{pmatrix} \tag{4.47}$$

There exist

$$H = \begin{pmatrix} 0 & S \\ S^\dagger & 0 \end{pmatrix} \tag{4.48}$$

and

$$U = \begin{pmatrix} U_1 & 0 \\ 0 & U_2 \end{pmatrix} \tag{4.49}$$

with U_1 and U_2 unitary, such that

$$V^T M V = \begin{pmatrix} \Delta_L & 0 \\ 0 & \Delta_R \end{pmatrix} \tag{4.50}$$

with Δ_L and Δ_R diagonal, real and positive, with

$$V = e^{iH} U \tag{4.51}$$

Indeed, let

$$S = -i M_D^* (M_R^*)^{-1} \tag{4.52}$$

and let U_1 and U_2 be the unitary matrices such that

$$\begin{aligned} \Delta_L &= U_1^T m_1 U_1 \\ m_1 &= -M_D M_R^{-1} M_D^T \end{aligned} \tag{4.53}$$

and

$$\begin{aligned} \Delta_R &= U_2^T m_2 U_2 \\ m_2 &= M_R + \frac{1}{2}(M_R^*)^{-1} M_D^\dagger M_D + \frac{1}{2} M_D^T M_D^* (M_R^*)^{-1} \end{aligned} \tag{4.54}$$

One can then predict neutrino masses and oscillation lengths from the parameters in the mass matrices, which are more or less known for M_D and

open to speculation for M_R. The flavor of these speculations, rather than a solid result, can be obtained with a couple of simplifications.

In the model at hand, asume first that $\tilde{d}/d = \tilde{e}/e = \tilde{f}/f = \tilde{g}/g = \xi$. This assumption would follow from an $SO(10)$ symmetry broken at a higher scale than the $SU(5)$ unification we are considering. Such further unification must be rejected in the light of the discussion on baryogenesis (see below). Let $x_i = r_i^{-1} = (j_i M_0)^{-1}$, and let σ be any of the σ_i.

The eigenmasses are then, in terms of the quark masses,

$$\left[\frac{m_u\sqrt{m_u m_c}}{2r_1}\xi^2,\ \frac{m_c\sqrt{m_u m_c}}{r_1}\xi^2,\ \frac{m_t^2}{r_2}\xi^2,\ r_1-\epsilon_1,\ r_1+\epsilon_1,\ r_2+\epsilon_2\right] \tag{4.55}$$

with

$$\epsilon_1 = \frac{(m_c\xi)^2}{2r_1} \tag{4.56}$$

and

$$\epsilon_2 = -\frac{(m_t\xi)^2}{r_2} \tag{4.57}$$

The 90% confidence-level limits on neutrino masses[70,71] improve with every new analysis of e^+e^- runs and yet, given their order of magnitude

$$\begin{aligned} m_{\nu_\mu} &< 250 \text{ KeV} \\ m_{\nu_\tau} &< 125 \text{ MeV} \end{aligned} \tag{4.58}$$

they provide only very weak constraints. Tighter ones can be obtained from Zeldovich's cosmological limit[72,73]

$$m_{\nu_e} + m_{\nu_\mu} + m_{\nu_\tau} < 100 \text{ eV} \tag{4.59}$$

implying

$$\begin{aligned} r_1 &< 10^4\,\text{GeV} \\ r_2 &< 10^8\,\text{GeV} \end{aligned} \tag{4.60}$$

so that

$$\begin{aligned} j_1 &< 10^{-7}\left(\frac{10^{11}}{M_0}\right) \\ j_2 &< 10^{-3}\left(\frac{10^{11}}{M_0}\right) \end{aligned} \tag{4.61}$$

where M_0 is measured in GeV $(10^9 \leq M_0 \leq 10^{12})$.[59]

The mixing scheme depends very heavily on the relative magnitudes of the various parameters. Let us assume that $r_1 = r_2 = x^{-1}$, and neglect d (*i.e.*, neglect the up-quark mass) from all expressions. Then

$$U_1 \simeq \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & \frac{e}{g} \\ 0 & -\frac{e}{g} & 1 \end{pmatrix} \tag{4.62}$$

where we have taken $\sigma_4 \simeq \sigma_5$ $(\Rightarrow f \simeq -e)$. The estimate

$$V_{22} \simeq 1 - \frac{1}{2}\xi^2 x^2 (e\sigma_4)^2 \tag{4.63}$$

can be identified with the cosine of the effective mixing angle for muon neutrino disappearance,

$$\alpha_{\nu_\mu} \simeq \xi x e \sigma_4 \simeq \frac{m_c}{r} < 10^{-8} \tag{4.64}$$

The other parameter relevant for neutrino oscillations is

$$\begin{aligned} \Delta M^2 &= \left| m^2_{\nu_\mu} - m^2_{\nu_X} \right| \\ &\simeq m^2_{\nu_\tau} \simeq \left(\frac{\xi m_t^2}{r}\right)^2 \\ &< 3 \times 10^6 \, \mathrm{eV}^2 \end{aligned} \tag{4.65}$$

where we have used $m_t \simeq 45\,\mathrm{GeV}$. Given that the bound on j_1 is weaker, the mixing angle could be brought up to about 10^{-4}. An increase in the mixing angle, however, would probably imply an increase in Δm^2, so the outlook is rather grim: neutrino oscillations seem to be at least two orders of magnitude away from currently imaginable detectability.[74]

The Full Superpotential

In order to spontaneously break $SU(5)$ at a scale M_X, and the anomalous Peccei-Quinn symmetry at a scale M_0 ($10^9 < M_0 < 10^{12}\,\mathrm{GeV}$, from the existence of red giants and from axions not closing the universe[59]), we add a term to the superpotential[11,12]

$$G_2 \;=\; \eta_1(\Sigma\Sigma' - M_X^2)\phi_3 \;+\; \eta_2(\phi_1\phi_2 - M_0^2)\phi_4 \;+\; \eta_3\Sigma^3 \;+\; \eta_4\Sigma'^3 \tag{4.66}$$

The $SU(5)$ content of the various fields introduced, as well as their charges under the global $U(1)$'s, are shown in Table II.

In order to give mass to color triplets while leaving the isodoublets light,[60,61] we introduce the term[11,12]

$$\begin{aligned} G_3 \;&= (\lambda_1 K + \lambda_2 K' + \lambda_3 K'')MU \;+\; (\beta_1 K + \beta_2 K' + \beta_3 K'')MU' \\ &\quad + (\gamma_1 H + \gamma_2 L)\Sigma V \end{aligned} \tag{4.67}$$

which, taking $M >> \langle\Sigma\rangle = M_X$, (a) gives mass to two combinations of the quintets K, K', K'' pairing them with the $\overline{\mathbf{5}}$ fields U and U', leaving one light quintet, and (b) gives mass to one quintet (or rather, a doublet and a triplet) formed with L and H.

Adding furthermore a term[11,12]

$$\begin{aligned} G_4 \;&= (\delta_1 H\Theta_1 + \delta_2 L\Theta_1 + \delta_3 J\Theta_2)\Sigma \;+\; \delta_4 M\Theta_1\Theta_2 \\ &\quad +\; (\rho_1 K\Theta_2' + \rho_2 K'\Theta_2' + \rho_3 K''\Theta_2' + \rho_4 J'\Theta_1')\Sigma' \;+\; \rho_5 M\Theta_1'\Theta_2' \end{aligned} \tag{4.68}$$

masses can be given to the color-triplet components of the two light quintets formed with G_3, pairing them with the color triplets in J and J'. We thus end up with four triplets with mass around the intermediate scales $\delta M_X^2/M$ and $\rho M_X^2/M$ (two each), and four massless doublets, which will acquire a mass $\sim M_W$ from spontaneous electroweak symmetry breaking. Notice that only two of the doublets (triplets) couple to matter at the tree level. From the

discussion on the renormalization group equations below, we shall obtain an expression for M_X. The upper and lower bounds on the triplet masses translate into the bounds

$$10^{-5} < \rho_i, \delta_i < 10^{-4} \tag{4.69}$$

The anomalous Peccei-Quinn symmetry X is not broken explicitly; hence, dimension five operators are suppressed.[11,12] The full matter superpotential

$$G = G_1 + G_2 + G_3 + G_4 \tag{4.70}$$

is anomaly-free.

The $\mathbf{Z}_3$ discrete symmetry D under which

$$\Sigma \xrightarrow{D} e^{\frac{2\pi i}{3}} \Sigma \tag{4.71}$$

prevents terms like $KJ'\Sigma$ or $HJ\Sigma$ from appearing in G_4. It effectively forces Σ to be a **75** instead of a **24**, thus allowing the missing multiplet mechanism implemented here to work. Charges under D are displayed in Table II. Familiarity is unfortunately not preserved in the full G, so it is perhaps useful to think of it as a low-energy quasisymmetry. No familons are to be expected in this model, although one could imagine more complicated versions of $G_2 + G_3 + G_4$ that would require them at some intermediate scale.

TABLE II. Field content and symmetries (vectorial, Peccei-Quinn, discrete $\mathbf{Z}_3$) of G.

Superfield	SU(5) irrep	V	X	D
H	$\overline{\mathbf{5}}$	2	-2	2
L	$\overline{\mathbf{45}}$	2	-2	2
K	$\mathbf{5}$	-2	-2	2
K'	$\mathbf{5}$	-2	-2	2
K''	$\mathbf{5}$	-2	-2	2
ϕ_1	$\mathbf{1}$	-10	-2	0
ϕ_2	$\mathbf{1}$	10	2	0
ϕ_3	$\mathbf{1}$	0	0	0
ϕ_4	$\mathbf{1}$	0	0	0
Σ	$\mathbf{75}$	0	0	1
Σ'	$\mathbf{75}$	0	0	2
U	$\overline{\mathbf{5}}$	2	2	1
U'	$\overline{\mathbf{5}}$	2	2	1
V	$\mathbf{50}$	-2	2	0
J	$\mathbf{5}$	-2	2	2
J'	$\mathbf{5}$	2	2	0
Θ_1	$\mathbf{50}$	-2	2	0
Θ_1'	$\mathbf{50}$	-2	-2	1
Θ_2	$\overline{\mathbf{50}}$	2	-2	0
Θ_2'	$\overline{\mathbf{50}}$	2	2	2

PROTON DECAY

We know[75,76,77] that all possible proton-decay[78] operators are of one of the forms below. Greek indices run from one to three and represent color, Latin indices from the middle of the alphabet run from one to two and represent weak isospin and Latin indices from the beginning of the alphabet run from one to three and represent family or generation. Chirality is indicated for definiteness:

$$\begin{aligned}
O^1_{abcd} &= (\overline{d}^c_{\alpha a L} u_{\beta b R})(\overline{l}^c_{icR} q_{j\gamma d L})\epsilon_{\alpha\beta\gamma}\epsilon_{ij} \\
O^2_{abcd} &= (\overline{q}^c_{i\alpha a R} q_{j\beta b L})(\overline{e}^c_{cL} u_{\gamma d R})\epsilon_{\alpha\beta\gamma}\epsilon_{ij} \\
O^3_{abcd} &= (\overline{d}^c_{\alpha a L} u_{\beta b R})(\overline{e}^c_{cL} u_{\gamma d R})\epsilon_{\alpha\beta\gamma} \\
O^4_{abcd} &= (\overline{q}^c_{i\alpha a R} q_{j\beta b L})(\overline{l}^c_{mcR} q_{n\gamma d L})\epsilon_{\alpha\beta\gamma}\epsilon_{ij}\epsilon_{mn}
\end{aligned} \tag{5.1}$$

Recall that, as usual, $\overline{\psi}^c_L \chi_R = \overline{\chi}^c_L \psi_R$, and notice that

$$O^2_{abcd} = O^2_{bacd} \tag{5.2}$$

The form of these operators is constrained by the requirement of gauge invariance under the standard $SU(3)\times SU(2)\times U(1)$. They arise dynamically from integrating away heavy particles (higgs fields or gauge bosons of broken symmetries). The effective Lagrangian for the fermionic components of matter fields contains, in our case, O^1 from $(\mathbf{FT})^2$ terms and O^2 from $(\mathbf{TT})^2$, with the family structure dictated by familiarity ($\mathcal{F}$) invariance. There are no O^3 or O^4 operators, though, because we do not allow $H-K$ nor $L-K$ mixing terms; *i.e.*, we suppress proton-decay operators involving higgsinos[28,29,30] –with a chirality-flipping mass insertion in their propagator– and gluinos.

To illustrate the point, consider a one-family toy-model with a Yukawa superpotential of the form

$$a\,FTH \;+\; b\,TTK \tag{5.3}$$

where F, T, H and K transform as $\overline{\mathbf{5}}$, $\mathbf{10}$, $\overline{\mathbf{5}}$ and $\mathbf{5}$ of $SU(5)$. Both H and K contain color (anti)triplet, isosinglet pieces. These couple both to lepto-quark and to diquark superfields, causing proton decay in two different channels. $F\overline{F}$ can go into $T\overline{T}$ via H exchange, and $T\overline{T}$ can go to $T\overline{T}$ via K exchange.

Because of the helicity structure required by the structure of chiral vertices, these processes correspond to "dimension-six" proton-decay operators, in which all four external lines are fermionic and the bosonic component of the triplet (from H or K) propagates through the internal line.

Were we to further allow the mixing term

$$\mu\,HK \tag{5.4}$$

then tripletinos (but not bosonic triplets) would mix, inducing the process squark + squark $\to$ antiquark + lepton in the channel $FT \to \overline{T}\,\overline{T}$. The engineering dimension of such a diagram, with two scalar and two fermionic external legs, is five. The initial state can be virtually produced from a quark-quark pair (in the nucleon) via gluino exchange. Since gluinos couple strongly, the amplitude for the whole process (gluino exchange and higgsino exchange) is essentially the same as for the dimension-five higgsino exchange subprocess. This amplitude crucially differs from the one for quark + quark $\to$ antiquark + lepton via higgs boson exchange, in that it involves a triplet fermion propagator (rather than a triplet bosonic one), so it is suppressed by the mass (rather than the mass squared) of the triplet superfield.

Proton decay thus proceeds faster through these higgsino-mediated diagrams (dimension five) than through higgs-boson mediated ones (dimension six) roughly by a factor of $(\frac{M_{Higgs}}{M_{proton}})^2$. If the theory contains dimension five operators, the mass of the supertriplet must be boosted above $\sim 10^{14}$ GeV (rather than above $\sim 10^{10}$ GeV) in order to prevent fast proton decay rates.[60,79,80,81,82] We can suppress the mixing term μHK at the cost of losing the usual baryogenesis mechanism, from the interference between $H \rightarrow TT$ at tree level after μ-mixing of H into K, and $H \rightarrow TT$ at one loop with T, F and $H - K$ in the internal propagators. Note that in this one-family toy model this interference is zero even with the mixing term because the product of Yukawa couplings is necessarily real.[83] (The chiral structure of the processes is what interests us now.) Baryogenesis seems to require an interaction among, and hence the existence of, different families!

The baryogenesis problem will be solved shortly, but we can foresee that since the higgs triplets will have to be rather light, the dangerous mixings must be suppressed. The action presented above does so naturally with two of the global $U(1)$'s, Peccei-Quinn and familiarity. Around the electroweak scale, nevertheless, supersymmetry-violating effects will induce such mixing, safely suppressed by $O(\frac{m_{\frac{3}{2}}}{m_{triplet}})$.

Proton Decay Lagrangian

The superpotential arising from G_1 (Eq.(4.1)) involving matter and color-triplet higgses is given below, where family is indicated explicity but color and Dirac indices have been dropped.

In four-component notation, $\psi\chi$ stands for

$$\psi_\alpha C^\alpha{}_\beta \chi^\beta = \psi_L^T C \chi_l = \chi_L^T C \psi_L \tag{5.5}$$

For calculational purposes, it is much easier to use two-component notation and the physically relevant left- and right-handed fields, substituting in all the previous expressions

$$
\begin{aligned}
\psi &\longrightarrow \psi_L \\
\psi^c &\longrightarrow \sigma_2 \psi_R^*
\end{aligned}
\tag{5.6}
$$

and transposing the first entry in $\psi\chi$ after the substitution:

$$
\begin{aligned}
G_{triplets} =& \frac{a}{\sqrt{2}} H_3 \left(u_1^c d_2^c + u_2^c d_1^c - \ell_{1i} q_{2j} \epsilon^{ij} - \ell_{2i} q_{1j} \epsilon^{ij} \right) \\
&+ \frac{b}{\sqrt{2}} H_3 \left(u_3^c d_3^c - \ell_{3i} q_{3j} \epsilon^{ij} \right) + \frac{c}{\sqrt{2}} L_3 \left(u_2^c d_2^c - \ell_{2i} q_{2j} \epsilon^{ij} \right) \\
&- 4dK_3 \left(u_1^c e_2^c + u_2^c e_1^c + q_{1i} q_{2j} \epsilon^{ij} \right) - 4gK_3 \left(u_3^c e_3^c + u_3 d_3 \right) \\
&- 4eK_3' \left(u_2^c e_2^c + u_2 d_2 \right) - 4fK_3'' \left(u_2^c e_3^c + u_3^c e_2^c + q_{2i} q_{3j} \epsilon^{ij} \right)
\end{aligned}
\tag{5.7}
$$

where $\ell = \begin{pmatrix} \nu \\ e \end{pmatrix}$, $q = \begin{pmatrix} u \\ d \end{pmatrix}$, all the triplets have canonical kinetic terms, and the Yukawa couplings a, b, c, d, e, f, g were given in terms of measured fermion masses and constrained doublet *vev*'s above.

Triplets coupling to the same matter in Eq. (5.7) mix,

$$
\begin{aligned}
\begin{pmatrix} H_3 \\ L_3 \end{pmatrix} &= \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} \Xi_{FT} \\ X \end{pmatrix} = \begin{pmatrix} \cos\alpha\, \Xi_{FT} + \cdots \\ -\sin\alpha\, \Xi_{FT} + \cdots \end{pmatrix} \\
\begin{pmatrix} K_3 \\ K_3' \\ K_3'' \end{pmatrix} &= \begin{pmatrix} \alpha_1^\dagger & \beta_1^\dagger & \gamma_1^\dagger \\ \alpha_2^\dagger & \beta_2^\dagger & \gamma_2^\dagger \\ \alpha_3^\dagger & \beta_3^\dagger & \gamma_3^\dagger \end{pmatrix} \begin{pmatrix} \Xi_{TT} \\ X' \\ X'' \end{pmatrix} = \begin{pmatrix} \alpha_1^\dagger\, \Xi_{TT} + \cdots \\ \alpha_2^\dagger\, \Xi_{TT} + \cdots \\ \alpha_3^\dagger\, \Xi_{TT} + \cdots \end{pmatrix}
\end{aligned}
\tag{5.8}
$$

Only Ξ_{FT} and Ξ_{TT} remain light, with masses M_{FT}, M_{TT} around the intermediate scale, light by unification standards. The α_i can be chosen to be all real, just as we did with the doublets when we were dealing with the fermion masses. Although we have used the same notation (α_i) for the entries of the mixing matrices of doublets and triplets, there is no reason why they should be the same.

The superpotential (5.7) thus becomes effectively

$$
\begin{aligned}
G_{triplets} &= \Xi_{FT}\left[\cos\alpha\frac{a}{\sqrt{2}}\left(u_1^c d_2^c + u_2^c d_1^c - \ell_{1i} q_{2j}\epsilon^{ij} - \ell_{2i} q_{1j}\epsilon^{ij}\right)\right.\\
&+ \cos\alpha\frac{b}{\sqrt{2}}\left(u_3^c d_3^c - \ell_{3i} q_{3j}\epsilon^{ij}\right)\\
&- \left.\sin\alpha\frac{c}{\sqrt{2}}\left(u_2^c d_2^c - \ell_{2i} q_{2j}\epsilon^{ij}\right)\right]\\
&- 4\Xi_{TT}\left[d\alpha_1\left(u_1^c e_2^c + u_2^c e_1^c + q_{1i} q_{2j}\epsilon^{ij}\right) + g\alpha_1\left(u_3^c e_3^c + u_3 d_3\right)\right.\\
&+ \left. e\alpha_2\left(u_2^c e_2^c + u_2 d_2\right) + f\alpha_3\left(u_2^c e_3^c + u_3^c e_2^c + q_{2i} q_{3j}\epsilon^{ij}\right)\right]
\end{aligned}
\tag{5.9}
$$

This is the expression from which we will compute the baryon asymmetry of the universe. Since Ξ_{FT} and Ξ_{TT} do not mix, we need not consider higgsinos. The interaction term in the Lagrangian involving the scalar components of the higgs superfields Ξ and the fermionic components of matter in (5.9) is exactly the same expression (5.9), with the usual even-R fields instead of superfields.

At energies much lower than the masses of Ξ_{FT} or Ξ_{TT} we can integrate away Ξ_{FT} and Ξ_{TT}. Keeping only terms with only one lepton field, we arrive at the following low-energy Lagrangian for proton decay, where all fields are still current and not mass eigenstates:

$$
\mathcal{L} = \sum\left(L_{abcd} < abcd >_\nu - L_{abcd} < abcd >_e + R_{abcd}[abcd]\right) \tag{5.10}
$$

The L's and R's are coefficients with indices in family space (a, b, c, $d \in \{1,2,3\}$). The various proton decay operators are, in the notation of (5.1),

$$
\begin{aligned}
O^1_{abcd} &= < abcd >_\nu \; - \; < abcd >_e\\
O^2_{abcd} &= [abcd] \; + \; [bacd]
\end{aligned}
\tag{5.11}
$$

so we are breaking up the $SU(2)$ doublets q and ℓ appearing in (5.1), since proton decay takes place at energies below the $SU(2)$ breaking scale.

Explicitly,

$$
\begin{aligned}
< abcd >_\nu &= \left(d^c_a u^c_b\right)^\dagger \left(\nu_c d_d\right) \\
&= \left(\overline{d}^c_{aL} u_{bR}\right) \left(\overline{\nu}^c_{cR} d_{dL}\right) \\
< abcd >_e &= \left(d^c_a u^c_b\right)^\dagger \left(e_c u_d\right) \\
&= \left(\overline{d}^c_{aL} u_{bR}\right) \left(\overline{e}^c_{cR} u_{dL}\right) \\
[abcd] &= \left(u^c_a d^c_b\right) \left(e_c u_d\right)^\dagger \\
&= \left(\overline{u}^c_{aR} d_{bL}\right) \left(\overline{e}^c_{cL} u_{dR}\right)
\end{aligned}
\tag{5.12}
$$

or, in two-component notation,

$$
\begin{aligned}
< abcd >_\nu &= -\left(u^T_{bR} d_{aR}\right) \left(\nu^T_{cL} d_{dL}\right) \\
< abcd >_e &= -\left(u^T_{bR} d_{aR}\right) \left(e^T_{cL} u_{dL}\right) \\
[abcd] &= -\left(d^T_{bR} u_{aR}\right) \left(e^T_{cL} u_{dL}\right)
\end{aligned}
\tag{5.13}
$$

TABLE III. Coefficients of baryon-number-violating four-Fermi operators in effective Lagrangian before electroweak breaking.

$abcd$	$2M^2_{FT} L_{abcd}$	$16^{-1} M^2_{TT} R_{abcd}$
1212	$-a^2 \cos^2 \alpha$	$d^2 \alpha_1^2$
1222	$ac \sin\alpha \cos\alpha$	$de\, \alpha_1 \alpha_2$
1223		$df\, \alpha_1 \alpha_3$
1233	$-ab \cos^2 \alpha$	$dg\, \alpha_1^2$
2222	$-c^2 \sin^2 \alpha$	$e^2 \alpha_1^2$
2223		$ef\, \alpha_2 \alpha_3$
2233	$bc \sin\alpha \cos\alpha$	$ge\, \alpha_1 \alpha_2$
2323		$f^2 \alpha_3^2$
2333		$gf \alpha_1 \alpha_3$
3333	$-b^2 \cos^2 \alpha$	$g^2 \alpha_1^2$

The coefficients L_{abcd} and R_{abcd} satisfy the identities

$$\begin{aligned} L_{abcd} &= L_{(ab)(cd)} = L_{(cd)(ab)} \\ R_{abcd} &= R_{(ab)(cd)} = R_{(cd)(ab)} \end{aligned} \tag{5.14}$$

and they are all zero except the ones in Table III and their symmetrizations according to (5.14).

Rotate now all the quark and lepton fields into the mass eigenstates, denoted by the additional subscript m, and related to the current eigenstates by a unitary transformation $\psi = V_\psi \psi_m$. The new proton decay operators have the same form as the old ones, but with massive fields in them. They are related to the old ones by four mixings, one for each field. Summing over the various neutrino flavors, indistinguishable in proton-decay experiments, and neglecting the see-saw coupling between ordinary neutrinos and massive right-handed ones, we can express the Lagrangian (5.10) in terms of mass eigenstates:

$$\mathcal{L} = \sum \left(L^\nu_{abd} < abd > + L^e_{abcd} < abcd > + R^e_{abcd}[abcd] \right) \tag{5.15}$$

The coefficients

$$\begin{aligned} L^\nu_{abd} &= \sum_{e,f,g,h} L_{efgh} (V^\dagger_{-\frac{1}{3}R})_a{}^e (V^\dagger_{\frac{2}{3}R})_b{}^f (V_{-\frac{1}{3}L})^h{}_d \\ L^e_{abcd} &= -\sum_{e,f,g,h} L_{efgh} (V^\dagger_{-\frac{1}{3}R})_e{}^a (V^\dagger_{\frac{2}{3}R})^f{}_b (V_{\ell L})^g{}_c (V_{-\frac{1}{3}L})_h{}^d \\ R^e_{abcd} &= \sum_{e,f,g,h} R_{efgh} (V_{\frac{2}{3}R})^e{}_a (V_{-\frac{1}{3}R})^f{}_b (V^\dagger_{\ell L})_c{}^g (V^\dagger_{\frac{2}{3}L})_d{}^h \end{aligned} \tag{5.16}$$

can be evaluated using symbolic manipulations on a computer, without much problem. The full formal expressions are long and unilluminating.

Renormalization of Proton Decay Operators

One last step must be taken to obtain the effective proton decay Lagrangian, useful to compute proton decay rates: we must renormalize[84] all

the (dimension six) operators down from unification to our energies.[75,77,80] Indeed, the expressions for the coefficients L^{ν}, L^{e}, R^{e} of the various operators are valid at the unification scale. The renormalization of the Yukawa couplings themselves is taken into account by using low-energy fermion masses, but the full operators undergo further renormalization at low-energies, by a multiplicative enhancement factor which can be calculated using dimensional regularization. We neglect $U(1)$ renormalization, assume exact supersymmetry above M_W, and renormalize with one-loop equations down to the charm mass. Below M_W, we consider only $SU(3)$ renormalization. These are all reasonable assumptions in light of the requisite accuracy.

Both operators O^1 (mediated by scalar H_3 and L_3) and O^2 (mediated by scalar K_3, K_3', K_3'') could mix under $SU(3)$ renormalization with their supersymmetric partners, the dimension five operators mediated by higgsinos with the same quantum numbers–were those operators allowed by the theory at all. They cannot, however, mix with any other operators under $SU(2)$ renormalization because under exact supersymmetry the $SU(2)$ gaugino is Majorana massless: were it to be exchanged between two external legs with $SU(2)$ quantum numbers, it would undergo a helicity flip to preserve the chiral structure of the vertices, which it cannot do because (as advertised) it is massless.[85]

Let us consider the spectrum[11] contributing to the renormalization-group equations between M_X (unification scale) and the low-energy world. We have three light fermion families (the singlets are heavy but irrelevant for group behavior), with the top-quark mass around 45 GeV, four light isodoublets, with masses around M_W, and four light color triplets with masses around $M_0 \simeq 10^{10}$ GeV. We assume all other higgs superfields have masses of at least $O(M_X)$.

If the coupling constant for the ith simple factor of $SU(5)$ is, to one loop, of the form

$$\alpha_i(\mu) \;=\; \frac{4\pi}{b_i \log\left(\mu^2/\Lambda^2\right)} \tag{5.17}$$

we can find the coefficients b_i from the following expressions, where n_q (n_ℓ, n_d, n_t) is the number of quark flavors (lepton flavors, isodoublets, color triplets) below the energy scale μ,

For nonsupersymmetric QCD, ($SU(3)$ below M_W),

$$b_3 \;=\; 11 - \frac{2}{3} n_q \tag{5.18}$$

For supersymmetric QCD, ($SU(3)$ above M_W),

$$b_3 \;=\; 3 \times 3 - \frac{1}{2}(2n_q + n_t) \;=\; 3 - \frac{1}{2} n_t \tag{5.19}$$

where the 2 comes from counting quarks and antiquarks.

For supersymmetric $SU(2)$,

$$b_2 \;=\; 3 \times 2 - \frac{1}{2}(n_q + n_\ell + n_d) \;=\; -\frac{1}{2} n_d \tag{5.20}$$

Finally, for supersymmetric $U(1)$,

$$b_1 \;=\; -\frac{1}{2}(n_q + n_\ell) - frac310 n_d \;=\; -6 - \frac{3}{10} n_d \tag{5.21}$$

As boundary conditions for the renormalization-group equations we use

$$\begin{aligned} \alpha_{em}^{-1}(M_W) &= 128.5 \\ \alpha_3^{-1}(M_W) &= 0.12^{-1} \;=\; 8.3(\pm 0.4) \end{aligned} \tag{5.22}$$

Recall also that

$$\begin{aligned} \alpha_2^{-1}(M_W) &= \alpha_{em}^{-1}(M_W) \sin^2\theta_w \\ &= \alpha_{em}^{-1}(M_W) x_w \\ \alpha_1^{-1}(M_W) &= \frac{3}{5}\alpha_{em}^{-1}(M_W)\cos^2\theta_w \\ &= \frac{3}{5}\alpha_{em}^{-1}(M_W)(1 - x_w) \end{aligned} \tag{5.23}$$

whereas the unification scale is defined by

$$\alpha \;=\; \alpha_1(M_X) \;=\; \alpha_2(M_X) \;=\; \alpha_3(M_X) \tag{5.24}$$

The one-loop renormalization equations are thus

$$\begin{aligned}
\alpha^{-1} - \alpha_2^{-1}(M_W) &= \frac{1}{2\pi}(-2)\log\left(\frac{M_X}{M_W}\right) \\
\alpha^{-1} - \alpha_1^{-1}(M_W) &= \frac{1}{2\pi}(-7.2)\log\left(\frac{M_X}{M_W}\right) \\
\alpha^{-1} - \alpha_3^{-1}(M_0) &= \frac{1}{2\pi}(1)\log\left(\frac{M_X}{M_0}\right) \\
\alpha_3^{-1}(M_0) - \alpha_3^{-1}(M_W) &= \frac{1}{2\pi}(3)\log\left(\frac{M_0}{M_W}\right)
\end{aligned} \tag{5.25}$$

Solving them, one easily finds[11,86]

$$\begin{aligned}
M_X &= 5.6 \times 10^{16}\ \text{GeV} \\
\alpha &= \alpha(M_X) \;=\; (19.60)^{-1} \\
x_w &= \sin^2\theta_w(M_W) \;=\; 0.237
\end{aligned} \tag{5.26}$$

with errors of about 5% arising mostly from uncertainites in $\Lambda_{\overline{MS}}$, where $\overline{MS}$ is the modified minimal substraction scheme in QCD. The rather high value for x_w is in excellent agreement with the most recent measurements of x_w in deep-inelastic neutrino scattering.[87,88]

Now for the actual renormalization of the $qqq\ell$ operators. Since we are neglecting $U(1)$ effects, all operators at hand get renormalized in the same way. Letting A be any of the coefficients L^ν, L^e, R^e in the effective Lagrangian, the enhancement factors relate A at one scale to A (and all other B with the same quantum numbers) at some other scale. Specifically,

$$A(m_p) \;\simeq\; A(m_c) \;\simeq\; E_2 E_{3ns} E_{3ss} A(M_X) \tag{5.27}$$

where we have neglected E_1 and the enhancement factors are

$$
\begin{aligned}
E_2 &= \left[\frac{\alpha_2(M_W)}{\alpha}\right]^{\gamma_2/b_2} \\
E_{3ns} &= \left[\frac{\alpha_3(m_t)}{\alpha_3(M_W)}\right]^{\gamma_3^{ns}/b_3^{(6)}} \times \left[\frac{\alpha_3(m_b)}{\alpha_3(m_t)}\right]^{\gamma_3^{ns}/b_3^{(5)}} \times \left[\frac{\alpha_3(m_c)}{\alpha_3(m_b)}\right]^{\gamma_3^{ns}/b_3^{(4)}} \\
E_{3ss} &= \left[\frac{\alpha_3(M_W)}{\alpha_3(M_0)}\right]^{\gamma_3^{ss}/b_3^{(0)}} \times \left[\frac{\alpha_3(M_0)}{\alpha}\right]^{\gamma_3^{ss}/b_3^{(4)}}
\end{aligned}
\tag{5.28}
$$

The γ's are the relevant eigenvalues[77] of the anomalous dimension matrices, which happen to be the same as those recently considered for the case of dimension five operators:[85]

$$
\begin{aligned}
\gamma_2 &= \frac{3}{2} \\
\gamma_3^{ns} &= 2 \\
\gamma_3^{ss} &= \frac{4}{3}
\end{aligned}
\tag{5.29}
$$

The β-function coefficients can be read of from (5.18), with $n_d = 4$. The superindex in E_{3ns} denotes n_q, and n_t in E_{3ss}.

With a top-quark mass of 45 GeV [and $\alpha_3(M_0) \simeq 17^{-1}$, $M_0 \simeq 10^{10.5}$ GeV], the enhancement factor is

$$
E = E_2 \times E_{3ns} \times E_{3ss} = 1.39 \times 1.26 \times 1.65 = 2.89 \pm 0.2 \tag{5.30}
$$

Proton Decay Rates

Armed with this low-energy potential, we can calculate the proton (and neutron) decay amplituyes into various channels. However, a nucleon at rest is hardly a system made up of three bound pointlike quarks, and although much effort has been made, using various techniques,[78] to produce reliable amplitudes in terms of the $qqq\ell$ operators, large uncertainties remain. We follow here the most straightforward approach.[89] For the phase space, we assume the

quarks to be non-relativistic and the antilepton extremely relativistic.[90] All nuclear effects are disregarded, and we concentrate on two-body $p \rightarrow m\bar{\ell}$ decays; that is, we ignore the pion pole contributions and all three-body decays. Proton and neutron wave functions are taken to be $SU(6)$ symmetric,[91,92] where $SU(6) \supset SU(2)_{\rm spin} \times SU(3)_{\rm flavor}$ is the light-cone spin-flavor symmetry.

Following Ref. 89, the lifetimes of each mode can be conveniently written in the generic form[11]

$$\tau = y \left[\frac{m}{10^{10}} \right]^4 \tag{5.31}$$

where m is the mass (in GeV) of the color triplet mediating the decay, M_{FT} if the outgoing antilepton is right-handed, and M_{TT} if it is left-handed. The y coefficients (in years) for the main decay modes are shown in Table IV. Their explicit dependence on the triplet mixing parameters is also shown, and since we are looking for an limit on M_{FT} and M_{TT}, the upper bound (188 GeV) for all doublet *vev*'s has been used, *i.e.*, the bounds in (4.46) have been saturated.

At 90% confidence level, the latest limits on the proton lifetime into strange modes are[6]

$$\begin{aligned} \tau\left(p \rightarrow \mu^+ K^0\right) &> 4 \times 10^{31} \text{years} \\ \tau\left(p \rightarrow \overline{\nu} K^+\right) &> 5 \times 10^{31} \text{ years} \end{aligned} \tag{5.32}$$

These limits can be translated into lower bounds on the masses of the color triplets mediating proton decay:

$$\begin{aligned} m_{FT} &> 0.75 \times \sqrt{|15c^2 - 2.8s^2 - 0.055sc|} \times 10^{10}\,\text{GeV} \\ m_{FT} &> 0.53 \times \sqrt{|3.2c^2 - 12.2s^2 - 20sc|} \times 10^{10}\,\text{GeV} \\ m_{TT} &> 0.32 \times 10^{10}\,\text{GeV} \\ &\times \sqrt{|-319\alpha_1^2 + 319\alpha_2^2 + 0.6\alpha_3^2 - 2.7\alpha_1\alpha_2 - 0.17\alpha_1\alpha_3 - 27.8\alpha_2\alpha_3|} \end{aligned} \tag{5.33}$$

where c, s stand for $\cos\alpha$, $\sin\alpha$, and α_1, α_2, and α_3 stand, respectively, for $\cos\alpha'$, $\sin\alpha'\cos\alpha''$, and $\sin\alpha'\sin\alpha''$. For any values of the mixing angles, the above expressions can only vary so much, so that a perhaps more useful result is

$$\begin{aligned} m_{FT} &= \left[(4-10)\frac{\tau_\mu K}{4\times 10^{31}\text{years}}\right]^{\frac{1}{4}} \times 10^{10}\,\text{GeV} \\ m_{TT} &= \left[(5-18)\frac{\tau_\nu K}{5\times 10^{31}\text{years}}\right]^{\frac{1}{4}} \times 10^{10}\,\text{GeV} \end{aligned} \tag{5.34}$$

where the lifetimes are normalized by the current limits, for years of use.

It follows that masses of the color triplets are bound below by

$$m_{FT}, m_{TT} > 2\times 10^{10}\,\text{GeV} \tag{5.35}$$

This lower bound on the triplet masses translates into a lower bound for the mass of the inflaton (Δ^2/M) from which triplets decay,[7,8,9,10,11,12,13] so we can extract a bound on Δ purely from underground detector results:

$$\Delta > 10^{-4.25} \tag{5.36}$$

Remarkably, proton decay experiments set a lower limit on Δ which is essentially the same as the lower limit (3.9). The upper limit (3.5) on Δ arising from cosmological considerations on gravitino abundance at the time of nucleosynthesis is very close, so Δ is (as repeatedly advertised all along) very tightly constrained.

It is perhaps amusing to extract more mileage from the relationship between the reheat temperature and the gravitino mass, Eq.(3.4). The bound (5.35) translates into the bound (5.36) because inflatons must decay into triplets in order to produce the baryon asymmetric universe we live in. The two inequalities (3.4) and (5.36) result in

$$\mu < 1.54\times 10^{-7} = 3.7\times 10^{11}\,\text{GeV} \tag{5.37}$$

Proton decay experiments have thus obtained an upper bound on the gravitino mass! In numbers the result is not so impressive:

$$m_{\frac{3}{2}} \; < \; 57 \text{ TeV} \tag{5.38}$$

Along the same lines, the lack of experimental confirmation for supersymmetry in accelerator experiments allows us to expect

$$m_{\frac{3}{2}} > 100\,\text{GeV} \tag{5.39}$$

from which it follows that

$$\Delta < 10^{-3.3} \tag{5.40}$$

hence

$$m_3 < 1.9 \times 10^{12}\,\text{GeV} \implies \tau_{\mu K} < 1.3 \times 10^{40}\text{years} \tag{5.41}$$

On the other hand, assuming supersymmetry is not found at the SSC, then the gravitino mass will be narrowly constrained between lower bounds from the SSC and upper bounds from upgraded proton decay experiments. If the largest conceivable Earth-based detectors[6] are built and work, the proton lifetime limit will improve to

$$\tau_{\mu K} > 5 \times 10^{34}\text{years} \tag{5.42}$$

Then

$$\Delta > 10^{-3.9} \tag{5.43}$$

and

$$m_{\frac{3}{2}} < 7 \text{ TeV} \tag{5.44}$$

which could be a useful constraint.

TABLE IV. Decay rates for two-body final states

Mode	Γ : Branching Rates (yr^{-1})
$e^+_R\pi^0$	$1.02\times10^{27}\lvert E\times L^e_{1111}\rvert^2$
$e^+_L\pi^0$	$1.02\times10^{27}\lvert E\times R^e_{1111}\rvert^2$
$e^+_RK^0$	$1.39\times10^{27}\lvert E\times L^e_{2111}\rvert^2$
$e^+_LK^0$	$1.39\times10^{27}\lvert E\times L^e_{1211}\rvert^2$
$\mu^+_R\pi^0$	$1.00\times10^{27}\lvert E\times L^e_{1121}\rvert^2$
$\mu^+_L\pi^0$	$1.00\times10^{27}\lvert E\times R^e_{1121}\rvert^2$
$\mu^+_RK^0$	$1.34\times10^{27}\lvert E\times L^e_{2121}\rvert^2$
$\mu^+_LK^0$	$1.34\times10^{27}\lvert E\times L^e_{1221}\rvert^2$
$\overline{\nu}_R\pi^+$	$2.05\times10^{27}\lvert E\times L^\nu_{111}\rvert^2$
$\overline{\nu}_RK^+$	$1.39\times10^{27}\lvert E\times L^\nu_{112}\rvert^2$
$\overline{\nu}_RK^{*+}$	$0.012\times10^{27}\lvert E\times(3L^\nu_{112}-2L^\nu_{211})\rvert^2$
$e^+\eta$	$0.20\times\Gamma[e^+\pi^0]$
$e^+\rho^0$	$0.057\times\Gamma[e^+\pi^0]$
$e^+\omega$	$0.49\times\Gamma[e^+\pi^0]$
e^+K^{*0}	$0.0088\times\Gamma[e^+K^0]$
$\mu^+\eta$	$0.20\times\Gamma[\mu^+\pi^0]$
$\mu^+\rho^0$	$0.045\times\Gamma[\mu^+\pi^0]$
$\mu^+\omega$	$0.38\times\Gamma[\mu^+\pi^0]$
$\overline{\nu}\rho^+$	$0.20\times\Gamma[\overline{\nu}\pi^+]$

BARYOGENESIS

The observed baryon asymmetry[2,93] of the universe,

$$n_{B-\overline{B}}/n_\gamma \sim 10^{-10} \tag{6.1}$$

is a quantification of two surprising facts. One, that no significant amounts of antimatter exist. Two, that in fact there is much more matter per unit entropy (per photon) than one would expect from an equilibrium evolution of the primeval "singularity." The generation of this baryon asymmetry, a process known as baryogenesis, requires three fundamental conditions.[94] First, baryon number must obvioulsy be violated. Next, thermal equilibrium must also be violated. Finally, C and CP must also not be conserved.

Baryogenesis may proceed after the inflationary phase of the evolution fo the universe, by allowing the matter sector of the superpotential G to contain higgs triplets which produce a non-zero baryon-number asymmetry δB per triplet-antitriplet decay, and such that the inflaton can decay into them preferentially (*i.e.*, $m_3 < m_\phi$).

Since the inflaton is never in equilibrium with other fields, we do not have to worry about producing enough triplets via a Boltzmann distribution.[24] Indeed, the inflaton oscillates into the heaviest fields around, and we can simply estimate[7,8,11,12] that all of the inflaton's energy is released into heavy triplets which quickly decay into radiation, reheating the bubble to a temperature T_R:

$$\begin{aligned}\frac{n_{B-\overline{B}}}{n_\gamma} &\simeq \delta B\frac{n_3}{n_\gamma} \simeq \delta B\frac{n_\phi}{n_\gamma}\\ &\simeq \delta B\frac{n_\phi}{\rho_\phi/T_R} \simeq \delta B\frac{T_R}{m_\phi}\\ &\simeq \delta B\frac{\Delta}{M}\end{aligned} \tag{6.2}$$

In this expression δB is the baryon number asymmetry produced per decay of an inflaton and an anti-inflaton. Given that the inflaton's branching ratio into color triplets is practically one, then δB is essentially the baryon-number asymmetry produced per triplet-antitriplet decay, which we calculate from the matter sector of the GUT superpotential. Clearly, G must contain triplets of mass a little lower than the inflaton's, with a rather high decay asymmetry

$$\delta B \sim 10^{-6} \tag{6.3}$$

At tree level, all $|\text{amplitudes}|^2$ are real, so CP violation can be produced perturbatively only at the one-loop level. We need one-loop decays which can interfere with tree-level decays to produce a net baryon asymmetry.[25,26,27,95,2]

Indeed, if a generic particle X (colored higgs, say) can decay into two channels with different baryon number, for instance

$$\begin{aligned} X &\longrightarrow \overline{q}\,\overline{q} \qquad (B_{\text{final}} = -\frac{2}{3}),\\ X &\longrightarrow q\,\ell \qquad (B_{\text{final}} = \frac{1}{3}),\end{aligned} \tag{6.4}$$

where q is a quark and ℓ a lepton (superfields), then the baryon asymmetry δB produced per decay of a pair of X and $\overline{X}$ is

$$\begin{aligned}\delta B &= \frac{\frac{1}{3}\Gamma(X\to q\ell) - \frac{2}{3}\Gamma(X\to \overline{q}\,\overline{q})}{\Gamma(X\to q\ell)+\Gamma(X\to\overline{q}\,\overline{q})} + \frac{-\frac{1}{3}\Gamma(\overline{X}\to \overline{q}\,\overline{\ell}) + \frac{2}{3}\Gamma(\overline{X}\to qq)}{\Gamma(\overline{X}\to\overline{q}\,\overline{\ell})+\Gamma(\overline{X}\to qq)}\\ &= \frac{\Gamma(\overline{X}\to qq) - \Gamma(X\to\overline{q}\,\overline{q})}{\Gamma(X\to \text{anything})}\\ &= \frac{\Gamma(X\to q\ell) - \Gamma(\overline{X}\to\overline{q}\,\overline{\ell})}{\Gamma(X\to\text{anything})}\end{aligned} \tag{6.5}$$

where we have used CPT invariance.

In general,

$$\delta B = \frac{1}{\sum \Gamma_i} \sum B_i (\Gamma_i - \overline{\Gamma}_i) \tag{6.6}$$

where the sum extends over all decay channels i of X, each with a net baryon number B_i.

Let

$$\Gamma(X \to q\ell) = |g_T f_T + g_L f_L|^2 \tag{6.7}$$

where the amplitude is the sum of a tree-level graph and a one-loop graph (disregarding higher orders in $\hbar$), with g the (product of) coupling constant(s) and f the dynamical factor, which depends on masses and external momenta and is the same, by CPT invariance, for X as for $\overline{X}$ decays. Then

$$\Gamma(\overline{X} \to \overline{q}\overline{\ell}) = \left|g_T^* f_T + g_L^* f_L\right|^2 \tag{6.8}$$

so that (6.5) becomes simply

$$\delta B = \frac{4\mathrm{Im}(f_T^* f_L)\mathrm{Im}(g_T^* g_L)}{\Gamma(X \to \text{anything})} \tag{6.9}$$

or, in the general case with more than two decay channels,

$$\begin{aligned} \delta B &= \frac{4\sum B_i \mathrm{Im}(f_{Ti}^* f_{Li})\mathrm{Im}(g_{Ti}^* g_{Li})}{\sum |g_{Ti} f_{Ti} + g_{Li} f_{Li}|^2} \\ &\simeq \frac{4\sum B_i \mathrm{Im}(f_{Ti}^* f_{Li})\mathrm{Im}(g_{Ti}^* g_{Li})}{\sum |g_{Ti} f_{Ti}|^2} \end{aligned} \tag{6.10}$$

The expression $\mathrm{Im}(f_T^* f_L)$ can be calculated using the Landau-Cutkosky rules for the interference diagram $f_T^* f_L$. Since at tree level all the phases disappear, we may choose couplings so that $f_T^* = f_T$. Then $\mathrm{Im}(f_T^* f_L) = f_T \mathrm{Im} f_L$. To evaluate this, all one has to do is modify the internal propagators in f_L separating initial from final states by putting them on shell.

All said and done, only the innermost line in the diagrams, where a higgs triplet is usually propagated, is allowed to be off-shell. We could add gauge interactions to these diagrams but (contrary to low-energy processes, such as proton decay) at the energies relevant for our dicussion, all gauge couplings are weak, so that renormalization of these operators can be neglected. All vertices are chiral, and even cubic if we are willing to introduce spurious fields to take into account mass terms.

The only way arrows (chirality) can arrange themselves in the interference diagrams is with mass insertions (chirality flips) in either the external line only, or else in two of the internal non-higgs lines. The origin of these constraints is, in supergraph language, the fact that interactions in the superpotential involve only chiral fields, so all three arrows point either into or out of any given vertex.

All the baryon asymmetry is generated from the higgs fields coupling to the $\mathbf{10}\cdot\mathbf{10}$ sector, for otherwise the product of couplings $g_T^* g_L$ is real. More particularly, $\delta B \neq 0$ arises from the interplay between the only two higgs superfields coupling to the same sector, both with non-diagonal (in family space) couplings. Remarkably, both higgs bosons and higgsinos can produce a baryon asymmetry upon decay.

Taking into account that $K_3'' = \alpha_3^\dagger \Xi_{TT}$, $K_3 = \alpha_1^\dagger \Xi_{TT}$, one can evaluate the interference and obtain the following asymmetry per decay:[11,12]

$$\delta B \;\simeq\; \frac{1}{32\pi^2}\frac{|\alpha_1|^2|\alpha_3|^2 \mathrm{Im}(\tilde{d}_1^* \tilde{g} \tilde{f}_1^* \tilde{f}_2^* j_1^* j_2)}{|\tilde{f}_1|^2+|\tilde{f}_2|^2+|\tilde{d}_1|^2+|\tilde{e}|^2+|\tilde{g}|^2} \times \left[\frac{M_0}{M_{TT}}\right]^2 \tag{6.11}$$

Given that the triplet must decay into neutrinos (right-handed ones), with masses given by (4.55), it is clear that

$$j_1 M_0,\; j_2 M_0 \;<\; M_{TT} \tag{6.12}$$

so we can write, assuming a hierarchy of Yukawa couplings and all phases to be of order unity,

$$\delta B \ < \ \frac{1}{32\pi^2}\frac{|\alpha_1|^2|\alpha_3|^2|\tilde{d}_1|\,|\tilde{f}_1|\,|\tilde{f}_2|}{|\tilde{g}|} \tag{6.13}$$

To estimate this, take $\alpha_1 \sim \alpha_3 \sim 1$, disregard all doublet mixings, and assume proportionality between tilded and untilded Yukawa couplings [via $SO(10)$]. The value thus obtained,

$$\delta B < \ 10^{-3}\frac{\sqrt{m_u m_c}m_c^2}{m_t\sigma^2} \ \simeq \ 10^{-10} \tag{6.14}$$

is much too low, so the presumed $SO(10)$-inpired proportionality between tilded and untilded Yukawa couplings is to be rejected. Indeed taking all Yukawa couplings in (6.13) to be of order 10^{-1}, δB is about 10^{-5}, an order of magnitude above the required value.

It is thus rather easy to produce the baryon asymmetry of the universe from triplet decays into right-handed neutrinos and ordinary quarks. The parameters to play with are those in the neutrino Dirac mass matrix, while making the right-handed Majorana masses as large as allowed, *i.e.*, smaller than the triplet mass M_{TT}. The rather large Yukawa couplings between $SU(5)$ singlets and quarks could be ruled out if neutrino oscillations were to be observed, as pointed out above.

Baryon number violation has been searched for with greatest care in underground detectors, to no avail. The rather strong limits on proton decay rates provide us with a lower limit on the mass of the baryon-number-violating triplets. The upper limit on this mass (assuming Yukawa couplings to be perturbative, *i.e.*, smaller than one) sets an upper limit on the lifetime of the proton, limit which unfortunately falls beyond the neutrino-induced background on Earth.

CONCLUSIONS

We have achieved to construct a model in which the two bounds on the mass of the color triplets (lower from proton-decay experiments, upper from the requirement of baryogenesis) are met. Good relationships between fermion masses and a reasonable Kobayashi-Maskawa matrix (including CP violation) have been incorporated into an anomaly-free grand-unified model.

We have also established a hierarchy of masses

$$m_{\text{rightneutrino}} < m_{\text{triplet}} < m_{\text{inflaton}} < m_{\text{O'Raifearton}}$$

at the intermediate scale, $\sim 10^{10}\,\text{GeV}$ This intermediate scale arises from a plurality of physical considerations associated with supersymmetry breaking, inflation, baryogenesis, and the breaking of the Peccei-Quinn symmetry.

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BIOGRAPHICAL SKETCH

Born a month into the decade of the sixties in México, the candidate developed an early interest in physics while riding the giant roller coaster in the park of Chapultepec. A total solar eclipse the same year of the massacre of Tlatelolco further kindled a desire to understand nature, without necessarily pretending to dominate it.

His discovery in 1970 of the Mediterranean sea, its waves, and its peoples did not distract him from academic endeavors until 1976, the first year after fascism (and after high-school), during which he decided to join the Universitat de Barcelona as a freshman in physics. That tumultuous and exciting first taste of academia ended when the candidate moved to New York, to witness the last year of the "human-rights" presidency.

Disoriented by the contradictions apparent in the streets of New York, the candidate joined, along with most of his close friends, the CCFRR neutrino collaboration. After a rather long winter at Fermilab, he quit experimental high energy physics, secured a Master of Arts from Columbia University, and joined the Particle Theory group at the University of Florida, under the academic guidance of Professor Pierre Ramond.

Since then (1983), his research interests have meandered from the basic mechanics of grand unification, to the interface of particle physics and astrophysics, to the more formal issues associated with string theories and their compactifications, paying nevertheless close attention to the possible phenomenological implications of mathematical ideas.

The charming rural setting has proven most conducive towards research activities in a field somewhat unrelated to social conditions. The candidate appreciates some sports but particularly dislikes football, and deeply deplores the racist atmosphere of the University of Florida, the most obvious exponent of which has perhaps been the obstinately reactionary stance taken by its Administration on the apartheid regime and the divestment issue.

The candidate expects to continue research on the forefront of particle physics at the inter-European research center CERN. Two years there seem to constitute the only certainty of his personal future, which nevertheless looks brighter than that of the world at large. He hopes that the mathematical study of nature can contribute to a greater understanding of all the peoples of the world in tolerance, peace, and justice.

I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.

P. Ramond 3-30-87
Pierre Ramond, Chairman
Professor of Physics

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R. D. Field 3/30/87
Richard D. Field, Co-Chairman
Professor of Physics

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James N. Fry 3/30/87
James N. Fry
Professor of Physics

I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.

Pradeep Kumar.
Pradeep P. Kumar
Professor of Physics

I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.

Pierre Sikivie
Professor of Physics

I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.

Thomas W. Simon

Thomas W. Simon
Professor of Philosophy

This dissertation was submitted to the Graduate Faculty of the Department of Physics in the College of Liberal Arts and Sciences and to the Graduate School and was accepted as partial fulfillment of the requirements for the degree of Doctor of Philosophy.

May 1987

Dean, Graduate School